Blind Distributed Detection in MIMO Networks

Arun Deshwal, Adarsh Patel
School of Computing and Electrical Engineering (SCEE), Indian Institute of Technology Mandi, Mandi, H.P., India
Email: s19027@students.iitmandi.ac.in, adarsh@iitmandi.ac.in

***Abstract*—This work considers a blind detection problem in the distributed multiple-input multiple-output (MIMO) networks, i.e., wireless sensor, co-operative, IoT networks, etc., without knowing the encoding schemes of users, or sensors, and channel state information (CSI) between users and fusion center (FC). Each user observes the source phenomenon, encodes their observations, and transmit to the FC using same time-frequency channel resource, i.e., coherent multiple-access channel (MAC). Exploiting encoding and decision vectors across users to be orthogonal, a novel maximum-likelihood (ML) criteria-based constrained alternating least squares (CALS) algorithm is proposed for the blind distributed detection problem in the MIMO networks. Finally, simulation results illustrate the efficacy of the proposed CALS detection algorithm.**

***Index Terms*—Distributed detection, Alternating Least Squares.**

I. Introduction

Evolving applications of the distributed detection [1] in the communication networks, namely Wireless sensor networks (WSN) [2], co-operative communication and IoT networks [3], etc., have led to its comprehensive development. Such networks have multiple sensors/ users that sense/ observe a phenomenon to send their observations to the fusion center (FC), referred to as reporting. Optimal detection at FC requires minimising the impairments from the observation and reporting links. Hence, an exact estimate of the CSI becomes essential, thereby increasing communication overhead. Also, the required bandwidth for reporting links increases with the users. Meanwhile, the recent developments in IoT networks require solving distributed detection problems under low bandwidth, minimal cooperation and overhead constraints, employing advanced techniques [4]. The distributed detection problem has been extensively studied. This paper focuses on the coherent MAC channel-based distributed detection with minimal information at the FC. A similar multi-user detection was given in [5], which linked the Canonical polyadic decomposition (CPD)/ PARAFAC (Parallel Factorization) to the blind detection in DS-CDMA. Orthogonality-constrained CPD receiver was introduced in [6], which was later extended for MIMO [7] to get constrained bi-linear ALS (CBALS) assuming the CSI, an orthogonal matrix, and code matrix known at the FC.

This work proposes a detection algorithm to detect the source phenomenon only knowing the observation and encoding statistics. Hence, this work uses orthogonal codes, an orthogonal symbol matrix, and flat-faded CSI over coherent MAC, with all three parameters unknown to form a tri-linear least squares problem to yield a blind distribute detection algorithm, namely the constrained ALS (CALS), at the FC. The CALS detection algorithm is useful for networks which require constrained bandwidth and low latency. Constrained bandwidth, as the communication channel for users is the same, i.e., coherent MAC and low latency, as sending pilot symbols for channel estimation is not required. The following section presents the system description.

II. System Model

Consider a network with U users each having a transmit antenna that observe the phenomenon at the source and send their

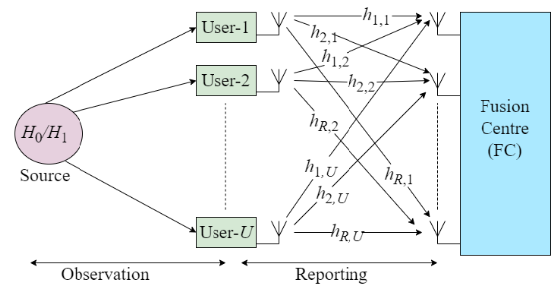

Fig. 1. A wireless user MIMO network observing the source phenomenon with U single antenna users and a FC with R receive antennas.

observations to the FC, having R receive antennas, as illustrated in Fig 1. The user u, $1 \leq u \leq U$, sense the source phenomenon and send the observation $\mathbf{d}_u \in \mathbb{R}^{K\times 1}$ to the FC over the wireless channel. The observation matrix $\mathbf{D}$ corresponding to U users can be written as $\mathbf{D} = [\mathbf{d}_1, \cdots, \mathbf{d}_u \cdots \mathbf{d}_U] \in \mathbb{R}^{K\times U}$. The observation vectors corresponding to the users are considered orthogonal, i.e., $\mathbf{D}^T\mathbf{D} = 2K\mathbf{I}_U$, where $\mathbf{I}_U$ denotes an identity matrix of size U, and $\mathbf{D}^T$ denotes the transpose of matrix $\mathbf{D}$. This is similar to considering the data of each user to be encoded, for instance, using pseudo-random noise (PN) codes. The phenomenon at the source, denoted by p, can take binary values to indicate the presence/ absence of the phenomenon corresponding to the alternative/ null hypothesis. The user u encode and send their observations to the FC over a wireless communication channel. The use of coherent multiple-access channel (MAC) between U users and R receive antennas of the FC drastically reduce the bandwidth requirement, multiple-user communication overhead, scheduling and synchronization problems. Let user u encodes the observation vector $\mathbf{d}_u$ using the code $\mathbf{s}_u$. The received signal $\mathbf{Y}_r \in \mathbb{C}^{K\times L}$ at antenna r of the FC corresponding to the transmission of U users sending their encoded observations be

$$\mathbf{Y}_r = \sum_{u=1}^{U} h_{r,u}\mathbf{d}_u\mathbf{s}_u^T + \mathbf{N}_r, \tag{1}$$

the channel coefficient matrix $\mathbf{H}$ contains the Rayleigh flat-faded channel coefficients between the U users and the FC given as $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_u, \cdots, \mathbf{h}_U] \in \mathbb{C}^{R\times U}$ where $\mathbf{h}_u$ denotes the channel coefficient vector between user u and FC, and an element $h_{r,u}$ of matrix $\mathbf{H}$, denotes the channel coefficient between user u and antenna r of the FC. Matrix $\mathbf{N}_r \in \mathbb{C}^{K\times L}$ denotes white Gaussian noise at antenna r of the FC. The element $n_{k,t}$ of the noise matrix $\mathbf{N}_r$ follows a zero mean Gaussian density with variance σ_n^2, i.e., $n_{k,t} \sim \mathcal{CN}(0, \sigma_n^2)$. The baseband system model (1) can be equivalently written as

$$\mathbf{Y}_r = \mathbf{D}\ \mathrm{Diag}([h_{r,1}, \cdots, h_{r,U}])\mathbf{S}^T + \mathbf{N}_r \tag{2}$$

$$= \sum_{u=1}^{U} \mathbf{d}_u \circ \mathbf{s}_u \circ h_{r,u} + \mathbf{N}_r, \tag{3}$$

where $\mathrm{Diag}([h_{r,1}, \cdots, h_{r,U}])$ in (2) is a diagonal matrix with its principle diagonal elements from row r of channel matrix

$\mathbf{H}$. The matrix $\mathbf{S}$ defines the coding scheme of U users as $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_u, \cdots, \mathbf{s}_U] \in \mathbb{R}^{L\times U}$, such that $\mathbf{S}^T\mathbf{S} = L\mathbf{I}_U$. The operator '$\circ$' in (3) denotes the outer product, also called as tensor product [8], defined between vectors $\mathbf{a} \in \mathbb{R}^{J_1}$ and $\mathbf{b} \in \mathbb{R}^{J_2}$ as $\mathbf{a} \circ \mathbf{b} = \mathbf{a}\mathbf{b}^T \in \mathbb{R}^{J_1\times J_2}$. The third-order tensor $\mathcal{Y} \in \mathbb{C}^{K\times L\times R}$ is obtained on stacking the $\mathbf{Y}_r$, given in (3), along the receive antennas from forward to backward [9], denoted as $\mathcal{Y} = [\mathbf{Y}_1|\mathbf{Y}_2|\cdots|\mathbf{Y}_R]$, is equivalently given as

$$\mathcal{Y} = \mathbf{D} \circ \mathbf{S} \circ \mathbf{H} + \mathcal{N} \tag{4}$$

$$= \sum_{u=1}^{U} \mathbf{d}_u \circ \mathbf{s}_u \circ \mathbf{h}_u + \mathcal{N}, \tag{5}$$

where the additive complex Gaussian noise tensor $\mathcal{N} = [\mathbf{N}_1|\mathbf{N}_2|\cdots\mathbf{N}_R] \in \mathbb{C}^{K\times L\times R}$ in (4) is obtained by stacking the noise matrices $\mathbf{N}_r, 1 \le r \le R$. The outer product of the observation matrix $\mathbf{D}$, encoding matrix $\mathbf{S}$, and the channel coefficient matrix $\mathbf{H}$, also called the factor matrices of the tensor $\mathcal{Y}$. The decomposition of U summation terms, popularly known as CANDECOMP/ PARAFAC (CP) decomposition (CPD) [10] is given in (5). Hence, (5) represents a third order rank-U tensor [11], which is decomposed into the sum of unique U rank-1 third order tensors, subject to the permutation and scaling/ sign ambiguity, under mild Kruskal rank condition [11]. The Kruskal rank-r of a matrix is the maximum number r such that every set of r columns are independent. For the factor matrices $\mathbf{D}$, $\mathbf{S}$, and $\mathbf{H}$, of the tensor $\mathcal{Y}$ in (5), the uniqueness condition required [11] is $k_{\mathbf{D}} + k_{\mathbf{S}} + k_{\mathbf{H}} \ge 2(U + 1)$. Where, $k_{\mathbf{D}}$, $k_{\mathbf{S}}$, and $k_{\mathbf{H}}$, denotes the Kruskal rank of the three factor matrices and U denotes the common dimension among the factor matrices. For full rank factor matrices the above Kruskal rank condition is reduced [5]

$$\min(K,U) + \min(L,U) + \min(R,U) \ge 2(U + 1). \tag{6}$$

Within the above stated challenges the next section presents novel detector to identify the source phenomenon without knowing codes of users and estimating their CSI in the network.

III. Blind Distributed Detector

Let the matrices $\hat{\mathbf{D}}$, $\hat{\mathbf{S}}$, and $\hat{\mathbf{H}}$ be the estimates of the factor matrices, corresponding to $\mathbf{D}$, $\mathbf{S}$, and $\mathbf{H}$, obtained from the received tensor $\mathcal{Y}$ in (5). To detect the source phenomena, first the estimate of the observation matrix $\hat{\mathbf{D}}$ from the received tensor is obtained and then an low complexity test is applied on the observation matrix. The optimal detection criterion for an equiprobable source symbols is the maximum likelihood (ML), which when applied to the system model in (5)/ (4) yields

$$\mathcal{L}(\hat{\mathbf{D}}, \hat{\mathbf{S}}, \hat{\mathbf{H}}) = \min_{\mathbf{D},\mathbf{S},\mathbf{H}} \left\| \mathcal{Y} - \sum_{u=1}^{U} \mathbf{d}_u \circ \mathbf{s}_u \circ \mathbf{h}_u \right\|_F^2 \tag{7}$$

$$= \min_{\mathbf{D},\mathbf{S},\mathbf{H}} \|\mathcal{Y} - \mathbf{D} \circ \mathbf{S} \circ \mathbf{H}\|_F^2 . \tag{8}$$

where $\|\cdot\|_F$ denotes Frobenius norm. The cost function in the above optimization framework (8) is nonlinear, in fact it is tri-linear. The above cost function (8) can be equivalently written as a set of three linear cost functions (9)-(11), presented next. The linear cost functions are obtained on reshaping the third-order tensor $\mathcal{Y} \in \mathbb{C}^{K\times L\times R}$ into matrices $\mathbf{Y}_{(1)} \in \mathbb{C}^{K\times LR}$, $\mathbf{Y}_{(2)} \in \mathbb{C}^{L\times RK}$, and $\mathbf{Y}_{(3)} \in \mathbb{C}^{R\times KL}$ called as mode-1, mode-2, and mode-3 matricization [8]. For a third-order tensor $\mathcal{Y}$, the mode-i for $i \in \{1, 2, 3\}$ matricization $\mathbf{Y}_{(i)}$ is obtained by stacking one of the three different types of matrix slices, called horizontal, lateral and frontal, i.e., when the indices k, l, and r are fixed. Hence, the tri-linear optimization framework for tensor $\mathcal{Y}$ in (8) when using the matricized mode-i, i.e.,$\mathbf{Y}_{(i)}$, for $i \in \{1, 2, 3\}$, can be equivalently converted into three cost functions as

$$\hat{\mathbf{H}} = \arg\min_{\mathbf{H}} \left\|\mathbf{Y}_{(3)} - \mathbf{H}(\mathbf{S} \odot \mathbf{D})^T\right\|_F^2 , \tag{9}$$

$$\hat{\mathbf{S}} = \arg\min_{\mathbf{S}} \left\|\mathbf{Y}_{(2)} - \mathbf{S}(\mathbf{H} \odot \mathbf{D})^T\right\|_F^2 , \tag{10}$$

$$\hat{\mathbf{D}} = \arg\min_{\mathbf{D}} \left\|\mathbf{Y}_{(1)} - \mathbf{D}(\mathbf{H} \odot \mathbf{S})^T\right\|_F^2 . \tag{11}$$

where the operator $\odot$ in (9)-(11) denotes the Khatri-Rao product [8]. The cost functions in (9)-(11) are linear, tractable, and have a closed form solution over the intractable tri-linear cost function in (8). The closed form expression to linear cost function (9) is

$$\hat{\mathbf{H}} = \mathbf{Y}_{(3)} \left((\mathbf{S} \odot \mathbf{D})^T\right)^{\dagger} , \tag{12}$$

where the operators $\mathbf{B}^{\dagger}$ denotes Moore-Penrose pseudo-inverse of matrix $\mathbf{B}$. Exploiting the property of the optimization variable, i.e., the code matrix as illustrated in Section II, the optimization problems in (10) is equivalently reposed as a constrained minimization problem, described as

$$\hat{\mathbf{S}} = \arg\min_{\mathbf{S}} \quad \left\|\mathbf{Y}_{(2)} - \mathbf{S}(\mathbf{H} \odot \mathbf{D})^T\right\|_F^2 \qquad \text{subject to} \quad \mathbf{S}^T\mathbf{S} = L\mathbf{I}_U. \tag{13}$$

Therefore using the orthogonality of the codes of different users, the cost function in (13) can be solved to

$$\begin{aligned}&\|\mathbf{Y}_{(2)} - \mathbf{S}(\mathbf{H} \odot \mathbf{D})^T\|_F^2 \\ &= \mathrm{Tr}\Big\{\mathbf{Y}_{(2)}^T\mathbf{Y}_{(2)} - (\mathbf{H} \odot \mathbf{D})\mathbf{S}^T\mathbf{Y}_{(2)} - \mathbf{Y}_{(2)}^T\mathbf{S}(\mathbf{H} \odot \mathbf{D})^T \\ &\quad + (\mathbf{H} \odot \mathbf{D})L\mathbf{I}_U(\mathbf{H} \odot \mathbf{D})^T\Big\}.\end{aligned} \tag{14}$$

Now, the cost function in (14) is minimized with the constraint $\mathbf{S}^T\mathbf{S} = L\mathbf{I}_U$. Further solving to make use of the constraint and the identity $\mathrm{Tr}(\mathbf{A}^T\mathbf{B} + \mathbf{B}^T\mathbf{A}) = 2\mathrm{Tr}(\mathbf{B}^T\mathbf{A})$ yields the optimal code matrix $\hat{\mathbf{S}}$, given as

$$\hat{\mathbf{S}} = \frac{1}{2}\Big(\mathbf{Y}_{(2)}((\mathbf{H} \odot \mathbf{D})^T)^{\dagger} + L(\mathbf{Y}_{(2)}^T)^{\dagger}(\mathbf{H} \odot \mathbf{D})\Big). \tag{15}$$

Similarly, optimization problem (11) when constrained by orthogonality of observation matrix, i.e., $\mathbf{D}^T\mathbf{D} = 2K\mathbf{I}_U$, is reposed as

$$\hat{\mathbf{D}} = \arg\min_{\mathbf{D}} \quad \left\|\mathbf{Y}_{(1)} - \mathbf{D}(\mathbf{H} \odot \mathbf{S})^T\right\|_F^2 \qquad \text{subject to} \quad \mathbf{D}^T\mathbf{D} = 2K\mathbf{I}_U. \tag{16}$$

Further, solving the constrained cost function leads to the argument minimization of $\mathrm{Tr}\Big\{\mathbf{Y}_{(1)}^T\mathbf{Y}_{(1)} - (\mathbf{H} \odot \mathbf{S}\mathbf{D}^T\mathbf{Y}_{(1)} - \mathbf{Y}_{(1)}^T\mathbf{D}(\mathbf{H} \odot \mathbf{S})^T + 2K(\mathbf{H} \odot \mathbf{S})\mathbf{I}_U(\mathbf{H} \odot \mathbf{S})^T\Big\}$, when solved along similar lines as (14), results into the optimal $\hat{\mathbf{D}}$, derived as

$$\hat{\mathbf{D}} = \frac{1}{2}\left(\mathbf{Y}_{(1)}((\mathbf{H} \odot \mathbf{S})^T)^{\dagger} + 2K\left(\mathbf{Y}_{(1)}^T\right)^{\dagger}(\mathbf{H} \odot \mathbf{S})\right). \tag{17}$$

The optimization frameworks in (10) and (11) equivalently reduced to the constrained minimization problem (13) and (16). The closed form solutions to the optimization problems in (9), (13), and (16) obtained corresponding to the mode-1, mode-2, and mode-3 matricized tensor $\mathcal{Y}$, are given in (12), (15) and (17), respectively. To obtain the solution to the nonlinear, or the tri-linear cost function (8), the set of linear cost functions in the

Algorithm 1 Blind Distributed Detection Algorithm

Input: $\hat{\mathbf{S}}^{(0)}$, $\hat{\mathbf{H}}^{(0)}$, $\hat{\mathbf{D}}^{(0)}$, and $\mathcal{Y}$.
Output: $\check{\mathbf{D}}$

Initialisation:
$i = 1$, I_{max}, Tolerance, and Error $= 0$

while $i < I_{max}$ && Error $>$ Tolerance **do**

$$\hat{\mathbf{H}}^{(i)} = \mathbf{Y}_{(3)} \left((\hat{\mathbf{S}}^{(i-1)} \odot \hat{\mathbf{D}}^{(i-1)})^T \right)^{\dagger}$$

$$\hat{\mathbf{S}}^{(i)} = \frac{1}{2}\left(\mathbf{Y}_{(2)} \left(\left(\hat{\mathbf{H}}^{(i)} \odot \hat{\mathbf{D}}^{(i-1)}\right)^T \right)^{\dagger} + L\left(\mathbf{Y}_{(2)}^T\right)^{\dagger} \left(\hat{\mathbf{H}}^{(i)} \odot \hat{\mathbf{D}}^{(i-1)}\right) \right)$$

$$\hat{\mathbf{D}}^{(i)} = \frac{1}{2}\left(\mathbf{Y}_{(1)} \left(\left(\hat{\mathbf{H}}^{(i)} \odot \hat{\mathbf{S}}^{(i)}\right)^T \right)^{\dagger} + 2K\left(\mathbf{Y}_{(1)}^T\right)^{\dagger} \left(\hat{\mathbf{H}}^{(i)} \odot \hat{\mathbf{S}}^{(i)}\right) \right)$$

$$\text{Error} = \left\| \mathbf{Y}_{(1)} - \hat{\mathbf{D}}^{(i)}(\hat{\mathbf{H}}^{(i)} \odot \hat{\mathbf{S}}^{(i)})^T \right\|_F^2$$

$i = i + 1$

end while

return $\check{\mathbf{D}} \leftarrow \hat{\mathbf{D}}^{(i)}$

optimization frameworks (9), (13), and (16) are alternately optimized, optimized one-by-one in a sequence, to converge within a tolerable error. The steps to obtain the observation matrix $\check{\mathbf{D}}$ is illustrated in the Algorithm 1. The solution to the optimization frameworks in (9), (13), and (16) at step i of the blind detection algorithm are denoted as $\hat{\mathbf{H}}^{(i)}$, $\hat{\mathbf{S}}^{(i)}$, and $\hat{\mathbf{D}}^{(i)}$. Hence the matrices $\hat{\mathbf{H}}^{(0)}$, $\hat{\mathbf{S}}^{(0)}$, and $\hat{\mathbf{D}}^{(0)}$ indicates the initialization of the respective matrices at the start of the algorithm, and can be assumed randomly. The decoded signal matrix $\check{\mathbf{D}}$, obtained by via the detection Algorithm 1, may have sign ambiguity [9]. Hence, a test statistic which combines the observations to decode the source phenomenon p from the decoded signal matrix $\check{\mathbf{D}}$ at the FC in the coherent-MAC based MIMO network is

$$\hat{p} = \frac{1}{K}\text{Tr}\left(\check{\mathbf{D}}^H \check{\mathbf{D}}\right) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \gamma, \tag{18}$$

where $\hat{p}$ is the phenomenon detected at the FC corresponding to the source phenomenon p, γ the decision threshold, and $\mathcal{H}_0$ and $\mathcal{H}_1$ are the null and alternative hypothesis. Next section presents the simulation results to validates the detection performance of the proposed blind detector.

IV. Simulation Results

Consider a network with $U = 8$ users, each with single transmit antenna, and a FC with $R = \{4, 8\}$ receive antennas. The code length $L = 16$, observation block size $K = 8$, binary phenomenon at the source, i.e., $p \in \{\sqrt{2}, 0\}$ with equal probability, the element $h_{r,u}$ of the channel coefficient matrix $\mathbf{H} \in \mathbb{C}^{R \times U}$ follow $h_{r,u} \sim \mathcal{CN}(0, 1)$. The initial conditions for the CALS detection algorithm, i.e., the matrices $\hat{\mathbf{S}}^{(0)}$, $\hat{\mathbf{D}}^{(0)}$, and $\hat{\mathbf{H}}^{(0)}$ are initialized with random values with $I_{max} = 30$, and Tolerance to 10^{-4}. Fig. 2 compares the bit error rate (BER) versus the signal-to-noise (SNR) performances of the proposed CALS detector in (18) for $R = \{4, 8\}$ with the alternate least squares (ALS) detection algorithm for $R = \{4, 8\}$, the energy detector (ED) for $R = 4$ and the Genie aided zero forcing detector ZF (Genie) for $R = 4$, i.e., ZF having perfect knowledge of the code matrix $\mathbf{S}$ and channel matrix $\mathbf{H}$ at the receiver, serves as a benchmark detector. The $\mathbf{D}$ from ALS and ZF (Genie) is

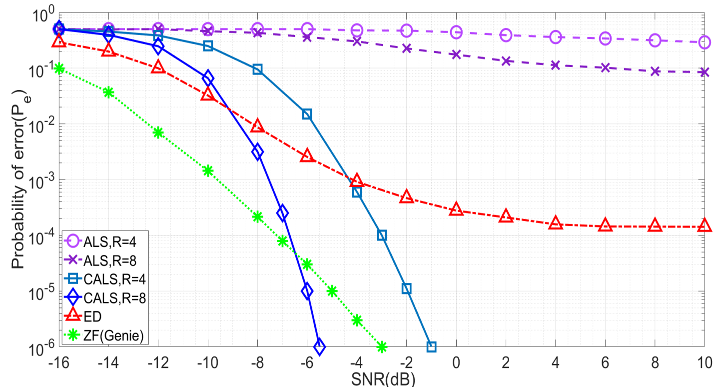

Fig. 2. BER vs. SNR simulation comparisons of the proposed CALS detection algorithm with ALS, ED and perfect information based ZF (Genie).

used in (18) to get the phenomenon. A superior performance of the proposed blind CALS detection algorithm is recorded over the other blind detection counterparts namely the ED and ALS. It is further noted that the performance of the proposed CALS improves with an increase in the receive antennas at the FC.

V. Conclusion

This work considered a distributed detection problem where each user observed the source phenomenon, encoded their decision and transmitted their information to the fusion centre over the wireless coherent MAC. This work presented an ML criteria-based constrained alternating least squares CALS algorithm that exploited the decision vectors' properties and encoding scheme to present an algorithm for the blind distributed detection problem in the MIMO networks. Finally, simulation comparisons illustrated the superior performance of the proposed algorithm.

References

[1] S.-T. Cheng, S.-Y. Li, and C.-M. Chen, "Distributed Detection in Wireless Sensor Networks," in *Seventh IEEE/ACIS International Conference on Computer and Information Science (icis 2008)*, pp. 401–406, 2008.

[2] M. Ayaz, M. Ammad-uddin, I. Baig, and e.-H. M. Aggoune, "Wireless sensor's civil applications, prototypes, and future integration possibilities: A review," *IEEE Sensors Journal*, vol. 18, no. 1, pp. 4–30, 2018.

[3] A. F. M. S. Shah and M. Shariful Islam, "A survey on cooperative communication in wireless networks," *International Journal of Intelligent Systems and Applications*, vol. 6, pp. 66–78, 06 2014.

[4] A. Cichocki, D. Mandic, L. De Lathauwer, G. Zhou, Q. Zhao, C. Caiafa, and H. A. PHAN, "Tensor decompositions for signal processing applications: From two-way to multiway component analysis," *IEEE Signal Processing Magazine*, vol. 32, no. 2, pp. 145–163, 2015.

[5] N. Sidiropoulos, G. Giannakis, and R. Bro, "Blind parafac receivers for DS-CDMA systems," *IEEE Transactions on Signal Processing*, vol. 48, no. 3, pp. 810–823, 2000.

[6] M. Sørensen, L. De Lathauwer, and L. Deneire, "Parafac with orthogonality in one mode and applications in ds-cdma systems," in *2010 IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 4142–4145, 2010.

[7] L. Zhao, S. Li, J. Zhang, and X. Mu, "A parafac-based blind channel estimation and symbol detection scheme for massive mimo systems," in *2018 International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery (CyberC)*, pp. 350–3503, 2018.

[8] T. G. Kolda, "Multilinear operators for higher-order decompositions.," tech. rep., Sandia National Laboratories (SNL), Albuquerque, NM, and Livermore, CA, 2006.

[9] N. D. Sidiropoulos, L. De Lathauwer, X. Fu, K. Huang, E. E. Papalexakis, and C. Faloutsos, "Tensor decomposition for signal processing and machine learning," *IEEE Transactions on Signal Processing*, vol. 65, no. 13, pp. 3551–3582, 2017.

[10] R. Harshman, "Foundations of the parafac procedure: Models and conditions for an" explanatory" multimodal factor analysis," *UCLA Working Papers in Phonetics*, 1970.

[11] A. Stegeman and N. D. Sidiropoulos, "On kruskal's uniqueness condition for the candecomp/parafac decomposition," *Linear Algebra and its Applications*, vol. 420, no. 2, pp. 540–552, 2007.